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# A PARTON MODEL FOR INCLUSIVE SEMILEPTONIC B MESON DECAYS

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## Abstract

The parton model for semileptonic B meson decays is studied with special attention to the decay distributions. We find that the spectra show dramatic variations when we introduce cuts on the hadronic energy or invariant mass of hadrons. Results for both  $b \rightarrow u$  and  $b \rightarrow c$  decays are presented. The detailed spectra may help to separate the two types of decays.

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The decays of B-mesons have been studied extensively and have been very useful in extracting properties of the weak interactions. In particular, the  $b \rightarrow u$  transition is still an active field of investigation. On the experimental side there is a large effort hindered by the difficulty of identifying the decays of B-mesons to light quarks. This difficulty of identifying  $b \rightarrow u$  events forced investigators to concentrate on the end-point spectrum of the electron energy or study exclusive decays[1]. In this letter we present an extended study of the semileptonic B-decays. We are interested in the decay

$$B \rightarrow X_u + e^- \bar{\nu} \quad (1)$$

for which we wish to give explicit distributions in several kinematic variables. With the decay spectrum completely determined by a single parameter in the distribution function, it may be compared to the competing process in  $B \rightarrow X_c + e^- \bar{\nu}$ . With the help of these distributions it should be possible to test characteristics of the decay products with various kinematical cuts, and, in particular, collect events typical for this process. The hope is that with the results presented here the experimental analyses could incorporate many more events than those confined to the endpoint region of the electron spectrum.

There are three models for the inclusive decays of reaction (1). The early model of Altarelli et al.[2], to be denoted as ACM, uses a spectator model and a distribution of quarks within the B-meson described by the Fermi motion of the spectator quark. This approach treats the phase space effects correctly, but is rather crude, as the authors state[2], because it depends on an unknown distribution function for the spectator quark. The model pictures the overall decay as the disintegration of the B-meson into the spectator quark plus the decay products of the heavy quark.

The second approach[3, 4] uses the parton model in an infinite momentum frame (IMF). The probability of finding a b-quark in a B-meson carrying a fraction  $x$  of the mesons momentum in the IMF is given by the distribution function  $f(x)$ . The distribution function  $f(x)$  has the functional form suggested by theoretical arguments[5, 6, 7] and it peaks at large values of  $x$ . The kinematics for the decay of a heavy to a light quark involve the correlation of two currents at short distances, where the incoherence of the decay products is justified. This model has recently been improved and applied

to double differential inclusive decay distributions in reference [8] where a detailed treatment of the kinematics and the allowed physical regions can be found. A special feature of the parton model is the distribution of b quark in the B-meson, which provides a continuum spectrum for the mass of the recoiling hadrons. Here we only summarize the main results and point out new features before turning to applications.

A third model, suggested very recently[9], improves the free quark model by including  $1/m_Q$  corrections from the heavy quark theory. These corrections turn out to be rather small. Obviously, the three models incorporate dynamical corrections in a different manner. In the parton model dynamic corrections are included through the experimentally determined distribution function and contributions from two scaling variables  $x_+$  and  $x_-$ , to be described below.

In the parton model[3, 4] the decay kinematics are different from those in deep inelastic scattering. As a consequence, the energy-momentum conserving  $\delta$ -function has now two roots, that is, it gives two scaling variables

$$x_{\pm} = \frac{q_0 \pm |\vec{q}|}{M_B} = \frac{q_{\pm}}{M_B} \quad (2)$$

which are the light-cone variables of the current-momentum. Thus we have again scaling of the distribution function but now in terms of two variables. We define the kinematic variables:  $P_B$  = momentum of B-meson;  $P_e, E_e$  = momentum, energy of electron;  $P_{\nu}$  = momentum of neutrino;  $P_X$  = momentum of hadrons;  $q = P_e + P_{\nu}$  = momentum of current; and  $M_X$  = invariant mass of the final hadronic system. In terms of these the decay is in general defined as:

$$d\Gamma = \frac{G^2 |V_{ub}|^2}{(2\pi)^5 M_B} L^{\mu\nu} W_{\mu\nu} \frac{d^3 P_e}{2E_e} \frac{d^3 P_{\nu}}{2E_{\nu}}. \quad (3)$$

The leptonic tensor has the simple form

$$L^{\mu\nu} = 2(P_e^{\mu} P_{\nu}^{\nu} + P_{\nu}^{\mu} P_e^{\nu} - g^{\mu\nu} P_e \cdot P_{\nu} + i\varepsilon^{\mu\nu}{}_{\alpha\beta} P_e^{\alpha} P_{\nu}^{\beta}). \quad (4)$$

The hadronic tensor is defined in analogy to deep inelastic scattering,

$$W_{\mu\nu}(P_B, q) = -g_{\mu\nu} W_1(q^2, q \cdot P_B) + \frac{P_{B\mu} P_{B\nu}}{M_B^2} W_2(q^2, q \cdot P_B) - i\varepsilon_{\mu\nu\alpha\beta} \frac{P_B^{\alpha} q^{\beta}}{M_B^2} W_3(q^2, q \cdot P_B) + \dots \quad (5)$$

where the integration over the hadronic variables has already been performed. After carrying out the integrations over  $x$  we arrive at the structure functions<sup>3</sup>

$$W_1(q^2, q \cdot P_B) = 2[f(x_+) - f(x_-)], \quad (6)$$

$$W_2(q^2, q \cdot P_B)/M_B^2 = \frac{4}{M_B |\vec{q}|} [x_+ f(x_+) + x_- f(x_-)], \quad (7)$$

$$W_3(q^2, q \cdot P_B)/M_B^2 = -\frac{2}{M_B |\vec{q}|} [f(x_+) + f(x_-)]. \quad (8)$$

and hence the  $b \rightarrow u$  triple differential decay rate

$$\frac{d\Gamma}{dE_e dq^2 ds} = \frac{G^2 |V_{ub}|^2}{8\pi^3 M_B} \frac{q_0 - E_e}{|\vec{q}|} \{x_+ f(x_+) (2E_e - M_B x_-) + (x_+ \leftrightarrow x_-)\}. \quad (9)$$

This formula is simple and shows the dependence on the two light-cone variables  $x_{\pm}$ .

The distribution function  $f(x)$  can be taken from references [5, 6, 7]. Its functional form is very similar to the fragmentation function of a  $b$  quark into a  $B$ -meson. It has also been argued, on physical grounds, that the distribution and fragmentation functions for heavy mesons are the same (reciprocity relation)[3, 10, 11]. The fragmentation function was measured in several experiments and can be represented[11, 12] as

$$f(x) = N \frac{x(1-x)^2}{[(1-x)^2 + \varepsilon_p x]^2} \quad (10)$$

with  $\varepsilon_p$  a parameter and  $N$  the normalization factor. We will use three values  $\varepsilon_p = 0.003, 0.006$ .

Finally, we must comment on the kinematic regions where the model is valid. Two criteria must be fulfilled.

(i) The decay involves the correlation of two currents. We are more confident in applying the model when this distance between the two currents is close to the light-cone or at short distances.

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<sup>3</sup>Note that the structure functions in the present article are different from those in [4].

(ii) The model should apply in the region where many particles are produced. Thus, we must avoid the edge of the phase space where one or two pions are produced.

To sum up, we feel rather confident in using the parton model for the decay  $B \rightarrow X_u + e^- \bar{\nu}$  away from the edges of the phase space. For the decay  $B \rightarrow X_c + e^- \bar{\nu}$ , we are less confident, because the distances involved are larger. Furthermore, for the decays  $B \rightarrow X_c + e^- \bar{\nu}$  we must keep the  $c$ -quark mass, which modifies equation (9), and it makes a difference whether we choose the minimum value of the final hadronic mass as  $m_c$  or  $M_D$ , as we will show in figures 1b and 2a.

As we mentioned, one of the purposes of this work is to present detailed spectra which can eventually be compared with experiments. The simplest parameter to measure is the electron energy. In addition one may be able to measure the total hadronic energy  $E_X = M_B - q_0$ . In Fig. 1a we show the  $b \rightarrow u$  double differential decay rate in  $E_e$  and  $q_0$ . The spectra show a striking dependence of  $q_0$ . Most of the events occur for  $0.3M_B \leq E_X \leq 0.5M_B$ . For smaller hadronic energies the spectra shift to higher electron energies,  $E_e \geq 2.0 \text{ GeV}$ . This correlation of events may help to isolate  $b \rightarrow u$  events.

For comparison we also calculated the distribution for the  $B \rightarrow X_c + e^- \bar{\nu}$  decay. In Fig. 1b we show the double differential decay rate in  $E_e$  and  $q_0$ . Comparing with Fig. 1a we note that this channel runs out of events at  $E_e = 2.25 \text{ GeV}$  and the events above  $2 \text{ GeV}$  are very few. By making a cut in  $q_0 > M_B - M_D = 0.65M_B$  there are no events left for  $b \rightarrow c$ . This can be used as another criterion for isolating  $b \rightarrow u$  events.

The integrated spectrum  $d\Gamma/dE_e$  is shown in Fig. 2a for two fragmentation parameters  $\varepsilon_p$ . In the same figure we show the corresponding spectra for  $b \rightarrow c$ . For these curves we have chosen  $|V_{cb}| = |V_{ub}|$ . It is clear that the  $b \rightarrow c$  spectrum is softer and a very small fraction lies at  $E_e > 2.0 \text{ GeV}$ . We show two spectra for the  $b \rightarrow c$  decays, by varying the minimum value of the final hadronic mass. We have chosen two values  $(M_X)_{min} = m_c = 1.5 \text{ GeV}$  and  $(M_X)_{min} = M_D = 1.86 \text{ GeV}$ . We notice that the total decay rate varies considerably when we vary  $(M_X)_{min}$ . We feel that the final quark mass should

be replaced by the running charm quark mass<sup>4</sup>. As a result, the choice of  $(M_X)_{min}$  influences the determinations of  $|V_{cb}|$ .

For comparison we calculated the  $b \rightarrow u$  spectrum  $d\Gamma/dE_e$  in the ACM model[2]. We show in Fig. 2b the ACM and parton spectra together. (In these spectra we include QCD radiative corrections which appear as a multiplicative factor, as described in [8] after the work of [2, 13].) We note that the parton spectrum is lower and falls off less steeply than in the ACM model. Consequently, the values for  $V_{ub}$  extracted from the end-point energy will be *larger* in the parton model with the experimental fragmentation function of eqn (10). Of course we can reproduce the free quark model by modifying the fragmentation function so that it has the limit  $\delta(1-x)$  when the width parameter tends to zero, but this is not what is being measured for the fragmentation function [11, 12, 7]. This means that the semileptonic rate of a free  $b$  quark is substantially greater than that of a  $b$  quark confined in a  $B$  meson. This is analogous to deep inelastic scattering, where moments of the distribution functions were measured to be smaller than one[14]. Thus the spectra in Fig. 2b differ not only in shape but also in overall scale.

The double differential  $b \rightarrow u$  decay rate  $d\Gamma/dE_e dM_X$  depends strongly on the invariant mass of the hadrons  $M_X$ . We show in Fig. 3  $d\Gamma/dE_e dM_X$  for various values of  $M_X$ . The decay spectrum is larger in the range  $0.2M_B \leq M_X \leq 0.4M_B$ . The characteristic feature again is that the spectra shift to larger values of  $E_e$  as  $M_X$  decreases. After integration over  $E_e$  the sum of the curves gives the mass distribution, which agrees with mass distributions published before[3] and is similar but not identical with the curves in ref. [15].

The parton model provides an interesting alternative for analysing inclusive semileptonic decays in important kinematical regions where the decay is dominated by short distance physics. We have produced decay spectra for a variety of interesting and physically accessible observables which probe the decay dynamics in a much more complete way than the simple electron spectrum. These spectra are obtained from a simple and compact formula (eq. (9)) based on a one parameter parton model. As high statistics data

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<sup>4</sup>This topic is still under investigation.

become available this model may play an increasingly important role in separating  $b \rightarrow u$  from  $b \rightarrow c$  decays and lead to a better determination of the  $V_{ub}/V_{cb}$ . In fact, the distribution function  $f(x)$  can be measured, in principle, in the  $b \rightarrow u$  decays and then be used as an input to calculate the  $b \rightarrow c$  spectra.

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## Figure Captions

1a. The  $b \rightarrow u$  double differential decay rate  $d\Gamma/dE_e dq_0$  vs  $E_e$  for various values of  $q_0$  in the rest frame of the B meson. We set  $m_u = 0$ ,  $M_\pi = 0$ ,  $M_B = 5.3 \text{ GeV}$ ,  $\alpha_s = 0$  and  $\varepsilon_p = 0.006$ .

1b. The  $b \rightarrow c$  double differential decay rate  $d\Gamma/dE_e dq_0$  vs  $E_e$  for various values of  $q_0$  in the rest frame of the B meson. For the solid lines we use  $m_c = 1.5 \text{ GeV}$ ,  $M_B = 5.3 \text{ GeV}$ ,  $(M_X)_{min} = M_D = 1.86 \text{ GeV}$ ,  $\alpha_s = 0$  and  $\varepsilon_p = 0.006$ . The dashed lines have the same parameters except for  $(M_X)_{min} = m_c = 1.5 \text{ GeV}$ . For  $q_0 = 0.4M_B$  the two curves coincide.

2a.  $d\Gamma/dE_e$  for semileptonic B meson decays in the rest frame of the B meson. The values for the parameters are  $m_u = 0$ ,  $m_c = 1.5 \text{ GeV}$ ,  $M_B = 5.3 \text{ GeV}$ ,  $M_\pi = 0$ ,  $M_D = 1.86 \text{ GeV}$ ,  $\alpha_s = 0$  and various values of  $\varepsilon_p$  as shown. For the  $b \rightarrow c$  decays the low and high lines correspond to  $(M_X)_{min} = M_D$  and  $(M_X)_{min} = m_c$ , respectively.

2b. The  $b \rightarrow u$  spectrum in the ACM model and the parton model. Both curves include QCD radiative corrections with  $\alpha_s = 0.24$ .

3. The  $b \rightarrow u$  double differential decay rate  $d\Gamma/dE_e dM_X$  vs  $E_e$  for various values of  $M_X$  in the rest frame of the B meson. The values for the parameters are the same as in Fig. 1a.

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